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THE GRAVITATIONAL-WAVE CONTRIBUTION TO CMB ANISOTROPIES

AND THE AMPLITUDE OF MASS FLUCTUATIONS FROM COBE RESULTS

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Abstract A stochastic background of primordial gravitational waves may substantially contribute, via the Sachs–Wolfe effect, to the large–scale Cosmic Microwave Background (CMB) anisotropies recently detected by COBE. This implies a *bias* in any resulting determination of the primordial amplitude of density fluctuations. We consider the constraints imposed on $n < 1$ ("tilted") power–law fluctuation spectra, taking into account the contribution from both scalar and tensor waves, as predicted by power–law inflation. The gravitational–wave contribution to CMB anisotropies generally reduces the required rms level of mass fluctuation, thereby increasing the linear *bias parameter*, even in models where the spectral index is close to the Harrison–Zel'dovich value $n = 1$. This "gravitational–wave bias" helps to reconcile the predictions of CDM models with observations on pairwise galaxy velocity dispersion on small scales.

Subject headings: cosmic background radiation — cosmology — early universe — galaxies: formation

The recent detection of large angular scale CMB anisotropies by the COBE satellite (Smoot *et al.* 1992) opens a window to the understanding of the physics of the early universe: in particular, it provides strong constraints on models for the origin of primordial perturbations. Inflation is probably the simplest and most motivated of such models: perturbations are generated in a causal way by zero–point quantum fluctuations which are then magnified by the accelerated universe expansion to cosmologically observable scales. The determination of the *rms* fluctuation amplitude consistent with the COBE measurements in the frame of various galaxy formation scenarios (e.g. Wright *et al.* 1992; Efstathiou, Bond & White, 1992; Schaefer & Shafi 1992) has however shown that a quite high fluctuation level is required, which, in the standard Cold Dark Matter (CDM) scenario causes excessive small–scale power. In fact, a relevant quantity for all galaxy formation scenarios is the linear *bias parameter*, defined as the inverse of the *rms* linear mass fluctuation on a sphere of 8 h^{-1} Mpc (h is the Hubble constant in units of 100 km sec^{-1} Mpc^{-1}; we take $h = 0.5$): the COBE results imply $b \approx 0.8$, for the scale–invariant ($n = 1$) CDM case. Low bias (i.e. more evolved) CDM models lead to better agreement with observations on large-scale flows (e.g. Bertschinger *et al.* 1990), but imply an excess of velocity dispersion on small scales when compared to observations on pairwise galaxy velocity dispersion in the CfA redshift survey (Davis & Peebles 1983). Also, the slope of the galaxy two–point function, determined in numerical simulations, becomes too steep. Both of these drawbacks can be alleviated by resorting to a velocity bias (e.g. Couchman & Carlberg 1992).

The above determination of b is however only valid if the large angular scale temperature anisotropies, detected by COBE, are totally due to density perturbations (scalar modes), which perturb the last–scattering surface via the Sachs–Wolfe effect (Sachs & Wolfe 1967). However, a stochastic background of primordial gravitational waves (tensor modes) originated during inflation also contributes to this effect. A rough estimate of the anisotropies originated by scalar perturbations is $(\frac{\delta T}{T})_S \sim \frac{1}{5}H\delta\varphi/\dot{\varphi}$, where φ is the inflaton field, $\delta\varphi$ its fluctuation and H the Hubble constant during inflation; these quan-

tities have to be evaluated at the time when the scales relevant to the large–scale CMB fluctuations crossed the Hubble–radius, i.e. about 60 e–foldings before the end of the inflationary expansion. The anisotropy originated by tensor perturbations is $(\frac{\delta T}{T})_T \sim \frac{\kappa}{2}\delta\varphi$, with $\kappa \equiv \sqrt{8\pi G}$. Thus, we can easily obtain an approximate measure of their relative contribution by $(\frac{\delta T}{T})_S/(\frac{\delta T}{T})_T \sim \frac{2}{5}H/\kappa\dot\varphi|_{HC}$. During inflation $H^2 = \frac{\kappa^2}{3}(V(\varphi) + \frac{1}{2}\dot\varphi^2)$, where $V(\varphi)$ is the effective inflaton potential. In many models, such as chaotic (Linde 1983) or new inflation (Linde 1982; Albrecht & Steinhardt 1982), a slow–rollover approximation holds, $\dot\varphi^2 \ll V(\varphi) \approx 3H^2/\kappa^2$; in these cases the contribution of tensor modes to $\delta T/T$ is much smaller than that due to scalar ones. However, in other models, such as power–law inflation (Abbott & Wise 1984a; Lucchin & Matarrese 1985), a slow–rollover approximation is not necessarily required and one can have $\dot\varphi^2 \sim V(\varphi)$. More in general, the minimal requirement on the inflaton dynamics is that it should lead to accelerated universe expansion, which implies $\dot\varphi^2/V(\varphi) \equiv \epsilon < 1$ and $(\frac{\delta T}{T})_S/(\frac{\delta T}{T})_T \sim \frac{2}{5}\sqrt{\frac{1}{6} + \frac{1}{3\epsilon}} > \frac{\sqrt{2}}{5}$. It should be pointed out, however, that the possibility to ascribe most of the $\delta T/T$ signal to gravitational waves (Krauss & White 1992) is restricted to models where the fluctuation spectrum is non–scale–invariant.

To be more specific, let us consider the power–law inflation case, which has the advantage of being fully analytically tractable. In this case the universe expansion factor reads $a(t) = a_\star[1 + (H_\star/p)(t - t_\star)]^p \sim t^p$ (where $a_\star$ and $H_\star$ refer to an arbitrary time $t_\star$ during inflation), with $p > 1$, and the inflaton field is assumed to have an exponential potential (Lucchin & Matarrese 1985), $V(\varphi) \propto \exp(-\lambda\kappa\varphi)$, with $0 < \lambda = \sqrt{2/p} < \sqrt{2}$. Such an exponential potential also describes the dynamics of extended inflation models (e.g. La & Steinhardt 1989; Kolb, Salopek & Turner 1990). Moreover, for any inflation model where one scalar field rolls down a smooth potential, the evolution during the small range of e–foldings relevant for large–scale CMB anisotropies can be approximated by a power–law; thus, our results have a quite general validity. In such a case, the resulting power–spectrum of density perturbations at Hubble–radius crossing is proportional to $k^{2\alpha-3}$,

with $\alpha = 1/(1-p) < 0$. A stochastic background of gravitational waves is also produced, with the same spectral behaviour at Hubble–radius crossing. In what follows we shall parametrize both of these spectra by the index $n \equiv 2\alpha + 1 = (p-3)/(p-1) < 1$, which for scalar modes (but not for tensor ones!) gives the spectral slope on constant time hypersurfaces before recombination, i.e. the so–called primordial spectral index. Note that, the limit $p \to \infty$, corresponding to the de Sitter case, gives $\alpha \to 0$, or $n \to 1$, i.e. the Harrison–Zel'dovich fluctuation spectrum. For a general value of p (or n) one can obtain the estimate $(\frac{\delta T}{T})_S/(\frac{\delta T}{T})_T \sim \frac{1}{5}\sqrt{\frac{2(3-n)}{1-n}} = \frac{\sqrt{2p}}{5}$; thus, for values of n not too far from unity, as required by the COBE results, we conclude that gravitational waves may make a significant contribution to large angular scale CMB anisotropies.

Let us now provide a more detailed analysis of the problem. We can perform the usual expansion of temperature fluctuations in spherical harmonics $\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta, \phi)$, where the multipole coefficients take independent contributions from both scalar and tensor modes: $a_{\ell m} = a_{S,\ell m} + a_{T,\ell m}$. Note that, even though we wrote both the scalar and tensor modes as being proportional to the same field fluctuation $\delta\varphi$, they actually refer to independent quantum field fluctuations, namely the inflaton and one polarization state of the graviton, which simply have the same rms value. The squared multipole amplitudes $a_\ell^2 \equiv \sum_m |a_{\ell m}|^2$ have expectation values $\langle a_\ell^2 \rangle = \langle a_\ell^2 \rangle_S + \langle a_\ell^2 \rangle_T$, coming from both scalar and tensor perturbations. In the simplest case that both the inflaton and the graviton fluctuations have random phases, the multipoles a_ℓ are Rayleigh distributed in $2\ell + 1$ "dimensions" (Abbott & Wise 1984b; Fabbri, Lucchin & Matarrese 1987), with *cosmic variance* $\sqrt{2/(2\ell+1)}\langle a_\ell^2 \rangle$. The result for the scalar case is, in a flat universe, $\langle a_\ell^2 \rangle_S = \frac{(2\ell+1)}{9} \int_0^\infty \frac{dk}{k} \Delta_\Phi(k) j_\ell^2(2k/H_0)$ (e.g. Bond & Efstathiou 1987), where $\Delta_\Phi(k) \equiv \frac{k^3}{2\pi^2} \mathcal{P}_\Phi(k)$ is the power per logarithmic wavenumber of the peculiar gravitational potential Φ and $\mathcal{P}_\Phi(k)$ its power–spectrum. In the power–law inflation case this relation can be analytically integrated (e.g. Fabbri, Lucchin & Matarrese 1987; Lyth & Stewart

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1992) to give

$$\langle a_\ell^2 \rangle_S = \frac{(2\pi)^4}{25} \frac{3-n}{1-n} G(2\ell+1) \left(\frac{H_0}{2}\right)^{n-1} \frac{C(n)\Gamma(3-n)\Gamma(\ell+(n-1)/2)}{2^{2-n}\Gamma^2(2-n/2)\Gamma(\ell-(n-5)/2)}, \quad (1)$$

where the factor $C(n) \equiv \frac{1}{\pi^4 2^{n+1} a_*^2}(\frac{2a_* H_*}{3-n})^{3-n}\Gamma^2(2-n/2)$ is related to the power-spectrum of the inflaton by $\mathcal{P}_\varphi(k) = 4\pi C(n)k^{n-1}$. The constants a_* and H_* could be easily related to physical observables by matching the inflationary kinematics to the subsequent radiation and matter dominated eras. For the gravitational–wave contribution the result is (e.g. Abbott & Wise 1984a; Fabbri, Lucchin & Matarrese 1986)

$$\langle a_\ell^2 \rangle_T = 144\,\pi^5 G(2\ell+1)\frac{(\ell+2)!}{(\ell-2)!}\left(\frac{H_0}{2}\right)^{n-1} C(n)\int_0^\infty dk\,k^{n-2}I_\ell^2(k), \quad (2)$$

where

$$I_\ell(k) \equiv \int_{k\eta_E/\eta_0}^k dy \frac{J_{\ell+1/2}(k-y)}{(k-y)^{5/2}}\frac{J_{5/2}(y)}{y^{3/2}},$$

and η_E and η_0 are the conformal time at the recombination and at the present epoch. In this case, the integration must be numerically performed. However, in the range of values of n of interest for us, there is a nice property of tensor multipoles (e.g. Fabbri, Lucchin & Matarrese 1987) which makes it possible to relate them to the scalar ones in a simple manner. By numerically integrating $\langle a_\ell^2 \rangle_T$ in Eq.(2) one can show that the ratio $D_\ell(n) \equiv \langle a_\ell^2 \rangle_T/\langle a_\ell^2 \rangle_S$, for $\ell > 2$, is independent of ℓ to a very good approximation, $D_\ell(n) \approx D(n)$. From the plots of Figure 1 this can be seen to hold in the spectral range $0.5 \lesssim n < 1$, with better than 10% accuracy. Figure 1 also shows that, for $n \lesssim 0.8$, gravitational waves give the main contribution to the multipoles, while density perturbations dominate for larger n values. The ratio of the tensor to the scalar contribution to the quadrupole is larger than that due to the higher order multipoles. In the high ℓ limit ($\ell \gg 1$) it is also possible to obtain an approximate asymptotic form for the tensor multipoles, along the lines of the computation by Starobinskii (1985) for $n = 1$. Using Eq.(12) of Starobinskii as an approximation for $I_\ell^2(k)$ we obtain

$$\langle a_\ell^2 \rangle_T \approx 288\pi^5 C(n)F(n)\left(\frac{H_0}{2}\right)^{n-1}\left(\ell+\frac{1}{2}\right)^{n-2}, \quad (3)$$

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with

$$F(n) \equiv \int_0^1 dx \left[\frac{2}{9\pi^2}(1-x^2)^{\frac{1-n}{2}} \left(1 - \frac{3}{2}x^2 - \frac{3}{4}(1-x^2)x \ln\left(\frac{1+x}{1-x}\right)\right)^2 + \frac{x^2}{8}(1-x^2)^{\frac{9-n}{2}} \right], \quad (4)$$

which shows the same asymptotic dependence on ℓ as the scalar multipoles. The numerical value of the ratio of the tensor to the scalar components is 5% larger than those plotted in Figure 1 for $\ell = 30$, for values of $n > 0.5$, and 10% larger, for values of n between 0.5 and 0. We can then write any $\ell > 2$ multipole in terms of the scalar contribution only, $\langle a_\ell^2 \rangle = \langle a_\ell^2 \rangle_S (1 + D(n))$. This makes it easier to obtain the bounds imposed by the COBE determination of the angular correlation function $C(\theta)$ of temperature fluctuations to the complete *scalar + tensor* case. After dipole and quadrupole subtraction, one has $C(\theta) = (1 + D(n)) \sum_{l>2} (\Delta T_\ell^2)_S W^2(\ell) P_\ell(\cos\theta)$, where $(\Delta T_\ell^2)_S = (T_0^2/4\pi)\langle a_\ell^2 \rangle_S$, with $T_0 = 2.735 \pm 0.006$ K the mean temperature of the CMB radiation (Mather *et al.* 1990) and $W(\ell) = \exp[-(1/2)(\ell(\ell+1)/17.8^2)]$ the appropriate filter function for the DMR experiment (Smoot *et al.* 1992). Thus considering also the gravitational–wave contribution does not affect the best–fit on the primordial spectral index $n = 1.15^{+0.45}_{-0.65}$ (Smoot *et al.* 1992, Wright *et al.* 1992). This corresponds to a limit on the scale factor expansion power $p > 5$ and on the scalar field potential coupling constant $\lambda < \sqrt{2/5}$. It affects, instead, the best–fit on the amplitude, since the total *rms*–quadrupole–normalized amplitude $Q_{rms-PS} = 16.3 \pm 4.6$ μK can now be written as

$$Q_{rms-PS} = (Q_{rms-PS})_S \sqrt{1 + D(n)}. \quad (5)$$

The normalization of primordial density fluctuations required to fit the COBE data gets modified by the same factor. In particular one gets $b(n) = b_0(n)\sqrt{1 + D(n)}$, where b_0 represents the same quantity calculated disregarding the gravitational–wave contribution to CMB fluctuations. Figure 2 shows the ratio b/b_0 as a function of n; notice that this "gravitational–wave bias" is independent of the galaxy formation scenario, i.e. of the transfer function. A very good fit is $b/b_0 \approx \sqrt{\frac{14-12n}{3-n}}$. The value of $b_0(n)$ in the

frame of the CDM scenario, taking into account the (best-fitted) amplitude and the errors given by the COBE results on $C(\theta)$, can be easily derived by properly scaling the fits by Vittorio, Matarrese & Lucchin (1988) [their Eqs.(23) and (24)]: one obtains $b_0(n) \approx 0.75 \times 10^{1.285(1-n)}(1 \pm 0.28)$. From the COBE detection of the $10°$ anisotropy, $\sigma_{Sky}(10°) = 30 \pm 5 \ \mu K$, we similarly get $b_0(n) \approx 0.82 \times 10^{1.285(1-n)}(1 \pm 0.17)$, in good agreement with Adams $et\ al.$ (1992). By taking the weighted (by inverse variance) average of the two determinations and considering the gravitational–wave correction we finally get

$$b(n) \approx 0.80 \sqrt{\frac{14 - 12n}{3 - n}} 10^{1.285(1-n)}(1 \pm 0.15). \tag{6}$$

As it is clear from Figure 1, gravitational waves give an even larger contribution to the COBE quadrupole detection, $Q_{rms} = 13 \pm 6 \ \mu K$. In such a case we obtain $\langle a_2^2 \rangle_T / \langle a_2^2 \rangle_S \approx \frac{13(1-n)}{3-n}$ and, with the CDM transfer function, $b(n) \approx 0.94 \sqrt{\frac{16-14n}{3-n}} \ 10^{1.285(1-n)}(1 \pm 0.46)$, which, because of the higher cosmic variance, is affected by quite a large error bar.

Let us consider some examples. From Eq.(6), by taking $n = 0.8$, we get $b \approx 2.1$; even a value of the spectral index quite close to the Harrison–Zel'dovich one, such as $n = 0.9$, involves a remarkable correction by gravitational waves ($\approx 23\%$), leading to the final estimate $b \approx 1.3$.

Let us also notice that, given the inflationary model, the COBE data also provide a $determination$ of the value of the Hubble constant at the time when the largest observable scale left the horizon, i.e. about 60 e–foldings before the end of inflation. For instance, for a power–law inflation with $p = 11$ ($n = 0.8$), $H_{60} \simeq 1.48 \times 10^{-4} m_P (1 \pm 0.28)$, with m_P the Planck mass.

An important result of the present analysis is then the possibility to increase the estimate of the bias level: this implies less evolution of the considered cosmological models, thus lowering the amplitude of pairwise velocities on small scales. The tilted ($n < 1$) CDM models considered here provide a natural solution to the lack of power on large scales and excess power on small scales of a CDM model with $n = 1$ and $b \approx 1$. These non–standard

CDM models have been analyzed by many authors. Vittorio, Matarrese & Lucchin (1988) showed that they imply better agreement with large–scale drifts and with the cluster-cluster correlation function. Tormen, Lucchin & Matarrese (1992; see also Tormen *et al.* 1992) explored in wider detail the advantages of these models in reproducing the large-scale peculiar velocity field traced by optically selected galaxy samples. A good fit of the angular correlation function of galaxies in the APM catalog is obtained by Liddle, Lyth & Sutherland (1992), with a $n \approx 0.5$ CDM model. More recently, Adams *et al.* (1992) have considered various cosmological constraints, while Cen *et al.* (1992) have run numerical simulations of $n \approx 0.7$ CDM models; however, they fix the normalization by fitting the COBE data without the tensor–wave contribution. This fact implies an overestimate of the small–scale power leading to a residual excess of pairwise velocity dispersion. The increase of the COBE determined biasing factor, resulting from our analysis, gives then an even stronger support to tilted CDM models.

While completing this Letter two preprints have circulated that report on independent work on similar problems in the frame of various inflationary models (Salopek 1992; Davis *et al.* 1992). By a best–fit of the correlation function, Salopek derives the lower bound $p \gtrsim 11$ ($n \gtrsim 0.8$) in order to get an acceptable biasing factor. The Davis *et al.* analysis is instead based only on the quadrupole amplitude. Our conclusions are fully consistent with their results.

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Figure captions

Figure 1 The ratio of the tensor to the scalar contribution to the CMB multipoles, up to order $\ell = 30$, as a function of the primordial spectral index n.

Figure 2 The gravitational–wave correction to the linear bias, $b(n)/b_0(n) = \sqrt{1 + D(n)}$, as a function of the primordial spectral index n.
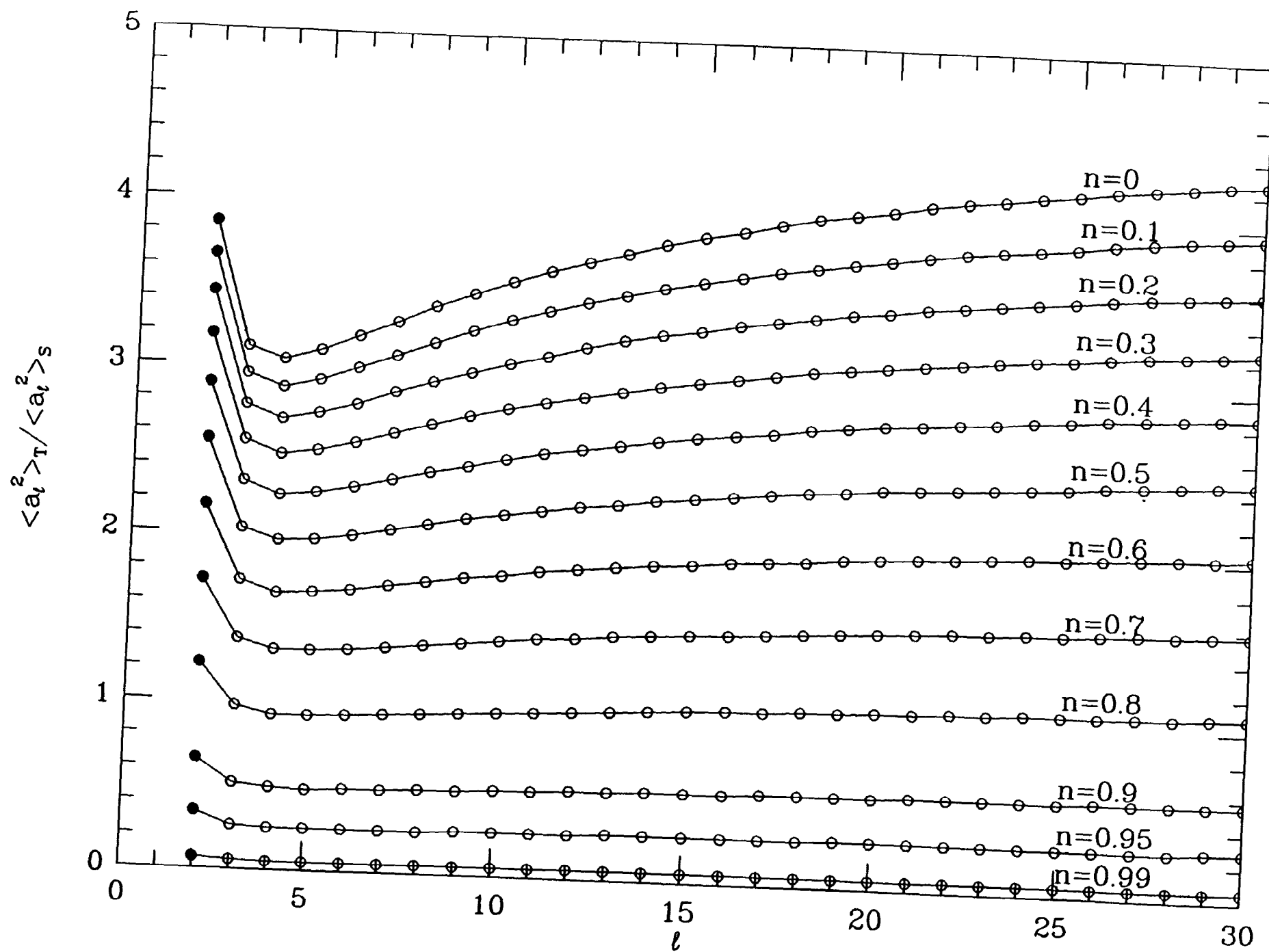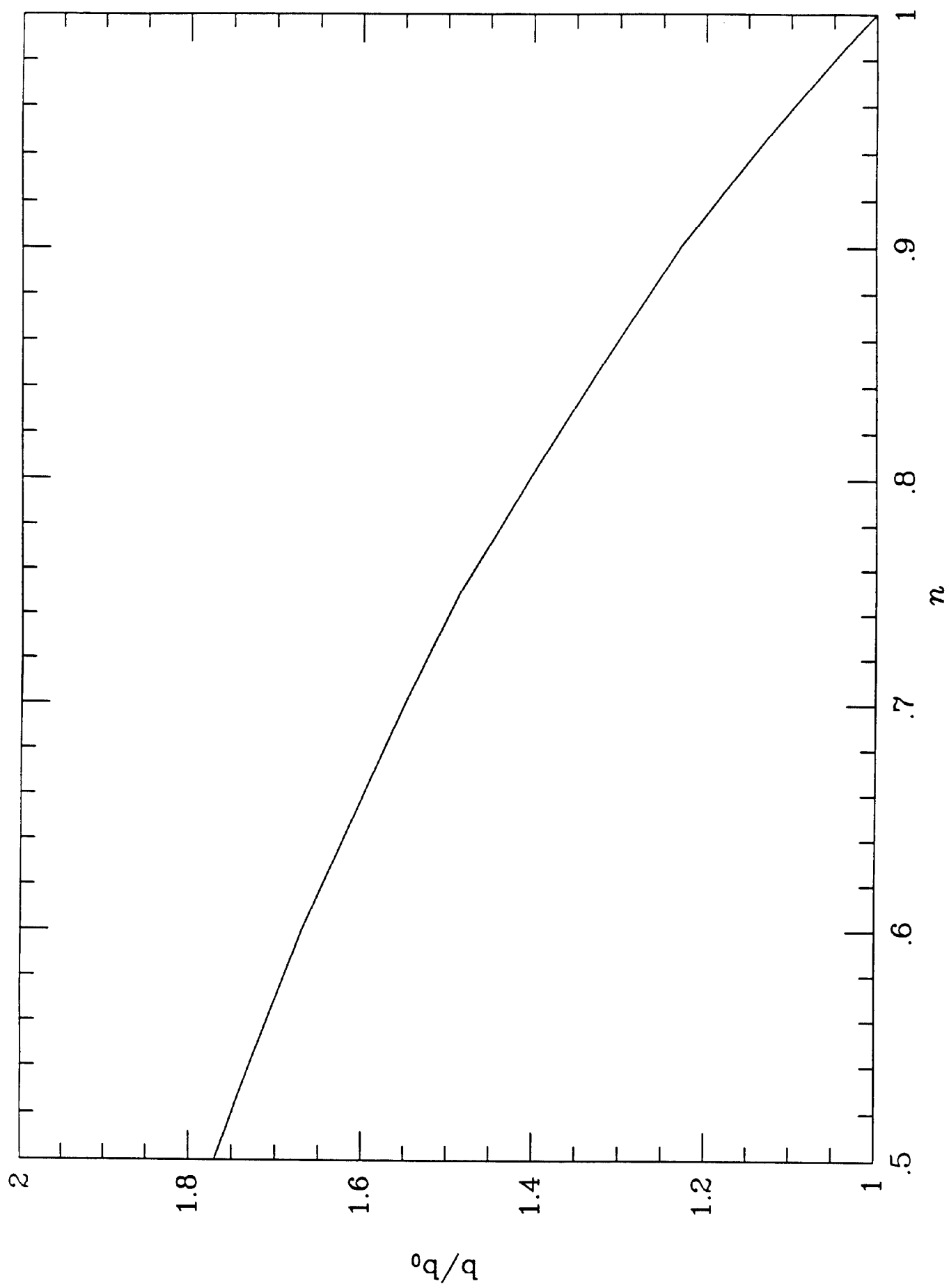
FIG. 1

FIG. 2